

Assignment - 2.

consistent and Inconsistent

Examine whether the following system of Equation are consistent if consistent solve.

1. $x + y + z = 6$

$$2x + 3y - 2z = 2$$

$$5x + y + 2z = 13$$

Sol:-
$$\left. \begin{array}{l} x + y + z = 6 \\ 2x + 3y - 2z = 2 \\ 5x + y + 2z = 13 \end{array} \right\} \text{given}$$

Equation to matrix form $[Ax=B]$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 5 & 1 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 2 \\ 13 \end{bmatrix}$$

matrix form. to augmented form matrix $[A:B]$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 5 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 2 \\ 13 \end{bmatrix}$$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 3 & -2 & 2 \\ 5 & 1 & 2 & 13 \end{array} \right]$$

Augmented matrix $\rightarrow$ Row Echelon form

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 2 & 3 & -2 & 2 \\ 3 & 1 & 2 & 13 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2 \times R_1$$

$$2 - 2(1) = 2 - 2 = 0$$

$$3 - 2(1) = 3 - 2 = 1$$

$$-2 - 2(1) = -2 - 2 = -4$$

$$2 - 2(6) = 2 - 12 = -10$$

$$R_3 \rightarrow R_3 - 3 \times R_1$$

$$3 - 3(1) = 3 - 3 = 0$$

$$1 - 3(1) = 1 - 3 = -2$$

$$2 - 3(1) = 2 - 3 = -1$$

$$13 - 3(6) = 13 - 18 = -5$$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & -10 \\ 0 & -2 & -3 & -17 \end{array} \right]$$

$$R_3 \rightarrow R_3 + 2 \times R_2$$

$$0 + 2(0) = 0$$

$$-4 + 2(-4) = -4 - 8 = -12$$

$$-3 + 2(-4) = -3 - 8 = -11$$

$$-17 + 2(-10) = -17 - 20 = -37$$

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & -10 \\ 0 & 0 & -12 & -37 \end{array} \right]$$

$$\text{Rank } A = 3, \text{ Rank } (A|B) =$$

$$\text{no. of variables} = 3$$

$$(i.e. = x, y, z)$$

$\text{Rank}(A) = \text{Rank}(A:B) = \text{no. of variables}$
 $\therefore$ The system of Equations are consistent
 and have unique solution.

solving
 matrix form $\rightarrow$ Equation form.

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & -10 \\ 0 & 0 & -19 & -52 \end{bmatrix}$$

$$x + y + z = 6 \rightarrow \textcircled{1}$$

$$y - 4z = -10 \rightarrow \textcircled{2}$$

$$-19z = -52 \rightarrow \textcircled{3}$$

from $\textcircled{3}$

$$-19z = -52$$

$$z = 3$$

substitute z in Equation $\textcircled{2}$

$$y - 4(3) = -10$$

$$y - 12 = -10$$

$$y = -10 + 12$$

$$y = 2$$

sub y, z in Equation $\textcircled{1}$

$$x + y + z = 6$$

$$x + 2 + 3 = 6$$

$$x + 5 = 6$$

$$x = 6 - 5 \cdot (1, 2, 3) \text{ which is } (1, 2, 3)$$

$$x = 1$$

$$\therefore x = 1, y = 2, z = 3$$

$$2. \quad 3x + 3y + 2z = 1$$

$$x + 2y = 4$$

$$10y + 3z = -2$$

$$2x - 3y - z = 5$$

$$\text{sol: } 3x + 3y + 2z = 1$$

$$x + 2y = 4$$

$$10y + 3z = -2$$

$$2x - 3y - z = 5$$

$$\begin{bmatrix} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{bmatrix}$$

given

Equation form $\rightarrow$ matrix form $[AX=B]$

$$A = \begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 4 \\ -2 \\ 5 \end{bmatrix}$$

matrix form $\rightarrow$ augmented matrix $(A|B)$

$$A = \begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 4 \\ -2 \\ 5 \end{bmatrix}$$

$$[A:B] = \begin{bmatrix} 5 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{bmatrix}$$

augmented matrix $\rightarrow$ row echelon form

$$= \begin{bmatrix} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$= \begin{bmatrix} 1 & 2 & 0 & 4 \\ 3 & 3 & 2 & 1 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{bmatrix} \quad R_2 \rightarrow R_2 - 3 \times R_1$$

$$3 - 3(1) = 3 - 3 = 0$$

$$3 - 3(2) = 3 - 6 = -3$$

$$2 - 3(0) = 2 - 0 = 2$$

$$1 - 3(4) = 1 - 12 = -11$$

$$R_4 \rightarrow R_4 - R_1 \times 2$$

$$2 - 2(1) = 2 - 2 = 0$$

$$-3 - 2(2) = -3 - 4 = -7$$

$$-1 - 2(0) = -1 - 0 = -1$$

$$5 - 2(4) = 5 - 8 = -3$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 10 & 3 & -2 \\ 0 & -7 & -1 & -3 \end{bmatrix} \quad R_3 \rightarrow R_3 \times 3 + R_2 \times 10$$

$$0(3) + 0(10) = 0 + 0 = 0$$

$$10(3) + (-3)(10) = 30 - 30 = 0$$

$$3(3) + 2(10) = 9 + 20 = 29$$

$$-2(3) + (-11)(10) = -6 - 110 = -116$$

$$R_4 \rightarrow R_4 \times 3 - R_2 \times 1$$

$$0(3) - 0(7) = 0 - 0 = 0$$

$$-7(3) - (-3)(7) = -21 + 21 = 0$$

$$-1(3) - 2(7) = -3 - 14 = -17$$

$$-3(3) - (11)(7) = -9 - 77 = -86$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & -17 & 68 \end{bmatrix}$$

$$R_4 \rightarrow R_4 \times 29 + R_3 \times 17$$

$$0(29) + 0 \times 17 = 0 + 0 = 0$$

$$0(29) + 0 \times 17 = 0 + 0 = 0$$

$$-17(29) + 29 \times 17 = 0$$

$$68(29) + (-116) \times 17 =$$

$$1972 - 1972$$

$$= 0$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Rank}(A) = 3$$

$$\text{Rank}(A:B) = 3$$

$$\text{no. of variables} = 3$$

$$\text{Rank}(A) = \text{Rank}(A:B) = \text{no. of variables}$$

$\therefore$ The system of Equations is consistent and have unique solution.

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 3 & 2 & -11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x + 2y = 4 \quad \text{--- (1)}$$

$$-3y + 2z = -11 \quad \text{--- (2)}$$

$$29z = -116 \quad \text{--- (3)}$$

from 3,

$$29z = -116$$

$$z = \frac{-116}{29}$$

$$z = -4$$

substitute z in Equation (2)

$$-3y = -8 + 11$$

$$-3y = 3$$

$$y = -1$$

sub y, z in Equation (1)

$$x + 2(-1) = 4$$

$$x + 2 = 4$$

$$x = 4 - 2 \quad x = 2$$

$$= \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{bmatrix} R_3 \rightarrow R_3 \times 7 + R_2 \times 6$$

$$0(7) - 6(0) = 0$$

$$-6(7) - 6(-7) = -42 + 42 = 0$$

$$5(7) - 6(5) = 35 - 30 = 5$$

$$(-4)(7) - 6(-8) = -28 + 48 = 20$$

$$R_4 \rightarrow R_4 \times 7 - R_2 \times 3$$

$$0(7) - 0(3) = 0$$

$$-3(7) - (-7)(3) = -21 + 21 = 0$$

$$2(7) - 5(3) = 14 - 15 = -1$$

$$-4(7) - (-8)(3) = -28 + 24 = -4$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & -1 & -4 \end{bmatrix} R_4 \rightarrow R_4 \times 5 - R_3$$

$$0(5) - 0 = 0$$

$$0(5) - 0 = 0$$

$$-1(5) - 5 = -5 - 5 = -10$$

$$-4(5) + 20 = -20 + 20 = 0$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Rank}(A) = 3$$

$$\text{Rank}(A:B) = 3$$

no. of variables = 3 (i.e. x_1, x_2, x_3)

$$\text{Rank}(A) = \text{Rank}(A:B) = \text{no. of variables}$$

$\therefore$ The system of Equation are consistent and have unique value.

matrix form $\rightarrow$ Equation form.

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow$$

$$x_1 + 2x_2 - x_3 = 3 \rightarrow \textcircled{1}$$

$$-7x_2 + 5x_3 = -8 \rightarrow \textcircled{2}$$

$$5x_3 = 20 \rightarrow \textcircled{3}$$

from Equation $\textcircled{3}$

$$5x_3 = 20$$

$$x_3 = 4$$

sub x_3 in Equation $\textcircled{2}$

$$-7x_2 + 5(4) = -8$$

$$-7x_2 + 20 = -8$$

$$2x_2 = 20 + 8$$

$$2x_2 = 28$$

$$x_2 = 14$$

substitute x_2, x_3 in Equation ①

$$x_1 + 2(14) \cdot 4 = 3$$

$$x_1 + 8 - 4 = 3$$

$$x_1 + 4 = 0$$

$$x_1 = -4$$

$$x_1 = -1$$

$$\therefore x_1 = -1, x_2 = 14, x_3 = 4$$

$$4) \quad x + y + z = 7$$

$$x + 2y + 3z = 8$$

$$y + 2z = 6$$

Sol.

$$x + y + z = 7$$

$$x + 2y + 3z = 8$$

$$y + 2z = 6$$

given Equations

Equation form to matrix form $[AX=B]$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 7 \\ 8 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 6 \end{bmatrix}$$

matrix form to Augmented matrix (A:B)

$$(A:B) = \begin{bmatrix} 1 & 1 & 1 & 7 \\ 1 & 2 & 3 & 8 \\ 0 & 1 & 2 & 6 \end{bmatrix}$$

augmented matrix $\rightarrow$ row Echelon form

$$(A:B) = \begin{bmatrix} 1 & 1 & 1 & 7 \\ 1 & 2 & 3 & 8 \\ 0 & 1 & 2 & 6 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$1-1=0$$

$$2-1=1$$

$$3-1=2$$

$$8-7=1$$

$$\begin{bmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 6 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$0-0=0$$

$$1-1=0$$

$$2-2=0$$

$$6-1=5$$

$$\begin{bmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$\text{Rank}[A] = 2$$

$$\text{Rank}[A:B] = 3$$

no. of variables = 3

$$\text{Rank}(A) \neq \text{Rank}(A:B)$$

$\therefore$ The system of Equations are inconsistent and have no solution.

$$\begin{aligned} 5Q) \quad 5x + 3y + 7z &= 4 \\ 3x + 26y + 2z &= 9 \\ 7x + 2y + 10z &= 5 \end{aligned}$$

Sol:-
$$\left. \begin{aligned} 5x + 3y + 7z &= 4 \\ 3x + 26y + 2z &= 9 \\ 7x + 2y + 10z &= 5 \end{aligned} \right\} \text{given Equation}$$

Equation to matrix form $[AX=B]$

$$A = \begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

matrix form $\rightarrow$ Augmented matrix $[A:B]$

$$A = \begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}_{3 \times 3}$$

$$B = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}_{3 \times 1}$$

$$(A:B) = \begin{bmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix}$$

augmented matrix $\xrightarrow{3 \times 4}$ Row Echelon form

$$\begin{bmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix} \begin{array}{l} R_2 \rightarrow R_2 \times 5 - R_1 \times 3 \\ 3(5) - 3(5) = 15 - 15 = 0 \\ 26(5) - 3(3) = 130 - 9 = 121 \\ 2(5) - 3(7) = 10 - 21 = -11 \\ 9(5) - 3(4) = 45 - 12 = 33 \end{array}$$

$$R_3 \rightarrow R_3 \times 5 - R_1 \times 7$$

$$7(5) - 7(5) = 35 - 35 = 0$$

$$2(5) - 7(3) = 10 - 21 = -11$$

$$10(5) - 7(7) = 50 - 49 = 1$$

$$5(5) - 7(4) = 25 - 28 = -3$$

$$\begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{bmatrix}$$

$$R_3 \rightarrow R_3 \times 11 + R_2$$

$$0 \times 11 + 0 = 0$$

$$-11(11) + 121 = -121 + 121 = 0$$

$$1(11) + (-11) = 11 - 11 = 0$$

$$-3(11) + 33 = -33 + 33 = 0$$

$$\begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \text{Rank}(A) = 2 \\ \text{Rank}(A:B) = 2 \end{array}$$

$$\text{no. of Variables} = 3$$

$$\text{Rank}(A) = \text{Rank}(A:B) \neq \text{no. of variables}$$

$\therefore$ system of Equation are consistent and have infinitely many no solution

then

$n - r$ arbitrary parameter in solution

$$3 - 2 = 1$$

1 arbitrary parameter in solution

consider z is arbitrary parameter, of let

$$z = k$$

$$\begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} 5x + 3y + 7z = 4 \rightarrow (1) \\ 121y - 11z = 33 \rightarrow (2) \end{array}$$

Sub z in (2)

$$121y - 11(k) = 33$$

$$121y = 33 + 11k$$

$$121y = 11(3 + k)$$

$$11y = 3 + k$$

$$y = \frac{3+k}{11}$$

Substitute y, z in Equation ①

$$5x + 3y + 7z = 4$$

$$5x + 3\left(\frac{3+k}{11}\right) + 7(k) = 4$$

$$55x + 9 + 3k + 77k = 44$$

$$55x + 80k = 35$$

$$5(11x + 16k) = 35$$

$$x = \frac{7-16k}{11}$$

$$\therefore x = \frac{7-16k}{11}, y = \frac{3+k}{11}, z = k.$$

6Q) $-x_1 + x_2 + 2x_3 = 2$

$$3x_1 - x_2 + x_3 = 6$$

$$-x_1 + 3x_2 + 4x_3 = 4$$

Sol:- $-x_1 + x_2 + 2x_3 = 2$

$$3x_1 - x_2 + x_3 = 6$$

$$-x_1 + 3x_2 + 4x_3 = 4$$

given

Equation $\rightarrow$ matrix form $[AX=B]$

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

matrix form $\rightarrow$ augmented matrix $CA: B$

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

$$A:B = \begin{bmatrix} -1 & 1 & 2 & 2 \\ 3 & -1 & 1 & 6 \\ -1 & 3 & 4 & 4 \end{bmatrix}$$

augmented matrix $\rightarrow$ Echelon form

$$\begin{bmatrix} -1 & 1 & 2 & 2 \\ 3 & -1 & 1 & 6 \\ -1 & 3 & 4 & 4 \end{bmatrix} \quad R_2 \rightarrow R_2 + R_1 \times 3:$$

$$3 + 3(-1) = 3 - 3 = 0$$

$$-1 + 3(1) = -1 + 3 = 2$$

$$1 + 3(2) = 1 + 6 = 7$$

$$6 + 3(2) = 6 + 6 = 12$$

$$R_3 \rightarrow R_3 - R_1$$

$$-1 - (-1) = -1 + 1 = 0$$

$$3 - (1) = 3 - 1 = 2$$

$$4 - (2) = 4 - 2 = 2$$

$$4 - (2) = 4 - 2 = 2$$

$$\begin{bmatrix} -1 & 1 & 2 & 2 \\ 0 & 2 & 7 & 12 \\ 0 & 2 & 2 & 2 \end{bmatrix} \quad R_3 \rightarrow R_3 - R_2$$

$$0 - 0 = 0$$

$$0 - 2 = -2$$

$$2 - 7 = -5$$

$$2 - 12 = -10$$

$$\begin{bmatrix} -1 & 1 & 2 & 2 \\ 0 & 2 & 7 & 12 \\ 0 & 0 & -5 & -10 \end{bmatrix}$$

$$\text{Rank}(A) = 3$$

$$\text{Rank}(A:B) = 3$$

$$\text{no. of variables} = 3$$

$$\text{Rank}(A) = \text{Rank}(A:B) = \text{no. of variables}$$

$\therefore$ The system of Equation is consistent and has unique value.

Solving the above matrix.

$$\begin{bmatrix} -1 & 1 & 2 & 2 \\ 0 & 2 & 7 & 12 \\ 0 & 0 & -5 & -10 \end{bmatrix} \begin{array}{l} \text{matrix form} \rightarrow \\ \text{Equation form.} \end{array}$$

$$-x_1 + x_2 + 2x_3 = 2 \rightarrow (1)$$

$$+2x_2 + 7x_3 = 12 \rightarrow (2)$$

$$-5x_3 = -10 \rightarrow (3)$$

from (3)

$$x_3 = 2$$

$$x_3 = 2$$

Sub x_3 in Equation (2)

$$-2x_2 + 7(2) = 12$$

$$-2x_2 = 14 - 12$$

$$-x_2 = -\frac{2}{2}$$

$$x_2 = -1$$

Sub x_2, x_3 in Eq ①

$$-x_1 + (-1) + 2(2) = 2$$

$$-x_1 - 1 + 4 = 2$$

$$x_1 = -1 + 4 - 2$$

$$x_1 = 1$$

70. find the values of a, b and b for which the system has (i) no solution (ii) unique solution and (iii) infinity many solutions

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + az = b$$

Sol:-

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + az = b$$

given

Equation $\rightarrow$ matrix form $[AX=B]$

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad b = \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix}$$

matrix form $\rightarrow$ Augment matrix (A|B)

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix}$$

$$(A|B) = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 2 \\ 2 & 3 & 0 & 6 \end{bmatrix}$$

augmented matrix $\rightarrow$ row reduction form

$$\begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 2 \\ 2 & 3 & 0 & 6 \end{bmatrix} \quad R_2 \rightarrow R_2 - R_1 \times 4$$

$$1(9) - 2(9) = 14 - 14 = 0$$

$$3(2) - 3(7) = 6 - 21 = -15$$

$$-2(2) - 5(9) = -4 - 35 = -39$$

$$8(2) - 9(9) = 16 - 53 = -47$$

$$R_3 \rightarrow R_3 - R_1$$

$$2 - 2 = 0$$

$$3 - 3 = 0$$

$$0 - 5 = -5$$

$$6 - 9 = -3$$

$$\begin{bmatrix} 2 & 3 & 5 & 9 \\ 0 & -15 & -39 & -47 \\ 0 & 0 & a-5 & b-9 \end{bmatrix}$$

i) no solution
if $\text{rank}(A) \neq \text{rank}(A:B)$

$$\text{Rank}(A) \rightarrow \boxed{\text{Rank}(A) < \text{rank}(A:B)}$$

The system of Equation has no solution

if $a=5$ and $b \neq 9$ the system of equation has no solution.

ii) unique solution

A system of Equation have unique solution when

$\text{rank}(A) = \text{rank}(A:B) = \text{no. of variables}$
when $a \neq 5$, b any value then the system of Equations have unique solution

iii) Infinity many solution.

A system of Equation has infinity many solutions when

$\text{rank}(A) = \text{rank}(A:B) < \text{no of variables } (n)$

when $a=5, b=5$ then the system of Equation has infinity many solution.

8. Discuss the solutions of the system of Equations for all values of λ

$$x+y+z=2$$

$$2x+y-2z=2$$

$$\lambda x+y+4z=2$$

Sol:-
$$\left. \begin{array}{l} x+y+z=2 \\ 2x+y-2z=2 \\ \lambda x+y+4z=2 \end{array} \right\} \text{given}$$

Equation form $\rightarrow$ matrix form $(AX=B)$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -2 \\ \lambda & 1 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -2 \\ \lambda & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

matrix form $\rightarrow$ Augmented matrix $(A:B)$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -2 \\ \lambda & 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$$(A:B) = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -2 & 2 \\ \lambda & 1 & 4 & 2 \end{bmatrix}$$

augmented matrix $\rightarrow$ row Echelon form

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -2 & 2 \\ \lambda & 1 & 4 & 2 \end{bmatrix} \quad R_2 \rightarrow R_2 - 2 \times R_1$$

$$2 - 2(1) = 2 - 2 = 0$$

$$1 - 2(1) = 1 - 2 = -1$$

$$-2 - 2(1) = -2 - 2 = -4$$

$$2 - 2(2) = 2 - 4 = -2$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -4 & -2 \\ \lambda & 1 & 4 & 2 \end{bmatrix} \quad R_3 \rightarrow R_3 + R_2$$

$$\lambda + 0 = \lambda$$

$$1 + (-1) = 0$$

$$4 + (-4) = 0$$

$$2 + (-2) = 0$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -4 & -2 \\ \lambda & 0 & 0 & 0 \end{bmatrix}$$

i) the system of Equation will be consistent and have unique solution if $\lambda \neq 0$

if $\lambda \neq 0$ Rank(A) = Rank(A:B) = $n = 3$

(ii) the system of Equation will be consistent and have

infinitely many no. of solution if $\lambda = 0$
 if $\lambda = 0$, $\text{Rank}(A) = \text{Rank}(A:B) = 2 < n(=3)$

90) $x + y + z = 6$
 $x + 2y + 3z = 14$
 $x + 4y + 7z = 30$

Sol:- $\left. \begin{array}{l} x + y + z = 6 \\ x + 2y + 3z = 14 \\ x + 4y + 7z = 30 \end{array} \right\} \text{given}$

Equation form $\rightarrow$ matrix form $[AX=B]$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

matrix $\rightarrow$ augmented matrix $(A:B)$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

augmented matrix $\rightarrow$ row Echelon form

$$(A:B) = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix} \quad \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$$

$$\begin{array}{l} 1-1=0 \\ 2-1=1 \\ 3-1=2 \\ 14-6=8 \\ 30-6=24 \end{array}$$

$$R_3 \rightarrow R_3 - R_2$$

$$1-1=0$$

$$4-3=1$$

$$7-3=4$$

$$30-14=16$$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix} \quad R_3 \rightarrow R_3 - R_2 \times 3$$

$$0-0(3)=0$$

$$3-1(3)=3-3=0$$

$$6-2(3)=6-6=0$$

$$24-8(3)=24-24=0$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Rank } A = 2 = \text{Rank } B$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Rank}(A) = \text{Rank}(A)$$

$$\text{Rank}(A) = \text{Rank}(A:B) < \text{no. of variables}$$

$\therefore$ The system of equations is consistent and have infinity many no. of solution

n-r orbitals

$$3-2=1$$

$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ matrix $\rightarrow$ Equation form

$$x+y+z=6 \rightarrow \textcircled{1}$$

$$y+2z=8 \rightarrow \textcircled{2}$$

consider z to be orbitals

consider $z=k$

sub z in Equation $\textcircled{2}$

$$y+2z=8$$

$$y+2(k)=8$$

$$y=8-2k$$

sub $y=8-2k$ in Eq. $\textcircled{1}$

$$x+(8-2k)+k=6$$

$$x+8-2k+k=6$$

$$x-k=6-8$$

$$x=\cancel{2}+k$$

$$x=1-k$$

$$\boxed{x=1-k, y=8-2k, z=k.}$$

$$\begin{aligned} 100) \quad & 2x - y + z = 7 \\ & 3x + y - 5z = 13 \\ & x + y + z = 5 \end{aligned}$$

sol:- $\left. \begin{aligned} 2x - y + z &= 7 \\ 3x + y - 5z &= 13 \\ x + y + z &= 5 \end{aligned} \right\} \text{ given}$

Equation $\rightarrow$ matrix form ($AX=B$)

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 7 \\ 13 \\ 5 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \\ 5 \end{bmatrix}$$

matrix $\rightarrow$ augmented matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 7 \\ 13 \\ 5 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 2 & -1 & 1 & 7 \\ 3 & 1 & -5 & 13 \\ 1 & 1 & 1 & 5 \end{bmatrix}$$

augmented matrix $\rightarrow$ row echelon form

$$= \begin{bmatrix} 2 & -1 & 1 & 7 \\ 3 & 1 & -5 & 13 \\ 1 & 1 & 1 & 5 \end{bmatrix} \quad R_3 \leftrightarrow R_1$$

$$R \begin{bmatrix} 1 & 1 & 1 & 5 \\ 3 & 1 & -5 & 13 \\ 2 & -1 & 1 & 7 \end{bmatrix} R_2 \rightarrow R_2 - 3R_1$$

$$3 - 3(1) = 3 - 3 = 0$$

$$1 - 3(1) = 1 - 3 = -2$$

$$-5 - 3(1) = -5 - 3 = -8$$

$$13 - 3(5) = 13 - 15 = -2$$

$$R_3 \rightarrow R_3 - R_2 \times 2 =$$

$$2 - 2(1) = 2 - 2 = 0$$

$$-1 - 2(-2) = -1 + 4 = 3$$

$$1 - 2(-2) = 1 + 4 = 5$$

$$7 - 2(-2) = 7 + 4 = 11$$

$$\begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 2 & -8 & -2 \\ 0 & -3 & -1 & -3 \end{bmatrix} R_3 \rightarrow R_3 \times 2 - R_2 \times 3$$

$$0(2) - 0 \times 3 = 0$$

$$-3(2) - (-8)(3) = -6 + 24 = 18$$

$$-1(2) - (-8)(3) = -2 + 24 = 22$$

$$-3(2) - (-2)(3) = -6 + 6 = 0$$

$$= \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 2 & -8 & -2 \\ 0 & 0 & 22 & 0 \end{bmatrix}$$

$$\text{Rank}(A) = 3$$

$$\text{Rank}(A:B) = 3$$

$$\text{no. of variables} = 3 \text{ (i.e., } x, y, z)$$

$\therefore \text{Rank}(A) = \text{Rank}(A:B) = \text{no of variables (n)}$
 $\therefore$ The system of Equations is consistent and have unique solution.

solving

matrix $\rightarrow$ Equation form

$$\begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & -2 & -8 & -2 \\ 0 & 0 & 22 & 0 \end{bmatrix} \begin{array}{l} x+y+z=5 \rightarrow (1) \\ -2y-8z=-2 \rightarrow (2) \\ 22z=0 \rightarrow (3) \end{array}$$

from (3)

$$22z=0$$

$$\boxed{z=0}$$

sub z in Eq (2)

$$-2y - 8z = -2$$

$$-2y - 8(0) = -2$$

$$y=1$$

sub y, z in Eq (1)

$$x+y+z=5$$

$$x+1+0=5$$

$$x=5-1$$

$$x=4$$

$$\therefore x=4, y=1, z=0$$

Solve the following system of Equations

11. $x+2y+3z=0$
 $2x+y+3z=0$

$$x+y+z=0$$

solⁿ $\left. \begin{array}{l} x+2y+3z=0 \\ 2x+y+3z=0 \\ x+y+z=0 \end{array} \right\} \text{ given}$

given Equation are homogeneous

Equation form $\rightarrow$ matrix form ($AX=0$)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A \cdot X = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

matrix $\rightarrow$ ~~augmented matrix~~ Echelon form

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1 \times 2$$

$$2 - 1(2) = 2 - 2 = 0$$

$$1 - 2(2) = 1 - 4 = -3$$

$$3 - 3(2) = 3 - 6 = -3$$

$$R_3 \rightarrow R_3 - R_1$$

$$1-1=0$$

$$1-2=-1$$

$$1-3=-2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -3 \\ 0 & -1 & -2 \end{bmatrix} R_3 \rightarrow R_3 \times 3 - R_2$$

$$0(3) - 0 = 0$$

$$-1(3) - (-3) = -3 + 3 = 0$$

$$-2(3) - (-3) = -6 + 3 = -3$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -3 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\text{Rank}(A) = 3$$

$$\text{no. of variables} = 3 \text{ (i.e. } x, y, z)$$

$$\text{Rank}(A) = \text{No. of variables}$$

∴ The system of Equation is consistent and have unique solution using

matrix $\rightarrow$ Equation form

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -3 \\ 0 & 0 & -3 \end{bmatrix} \begin{matrix} x + 2y + 3z = 0 \text{ --- (1)} \\ -3y - 3z = 0 \text{ --- (2)} \\ -3z = 0 \text{ --- (3)} \end{matrix}$$

from ③

$$-3z = 0$$

$$z = 0$$

sub z in Equation ②

$$-3y - 3(0) = 0$$

$$-3y - 0 = 0$$

$$-3y = 0$$

$$y = 0$$

$$\therefore x = y = z = 0$$

sub y & z in Equation ①

$$x + 2y + 3z = 0$$

$$x + 2(0) + 3(0) = 0$$

$$x + 0 + 0 = 0$$

$$x = 0$$

10) $x + y + 3z = 0$

$$x - y + z = 0$$

$$x - 2y = 0$$

sol.

$$x + y + 3z = 0$$

$$x - y + z = 0$$

$$x - 2y = 0$$

} given

Equation to matrix form $[Ax = 0]$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & -2 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

matrix form $\rightarrow$ Echelon form

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & -2 & 0 \end{bmatrix} \begin{matrix} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{matrix}$$

$$\begin{matrix} 1 - 1 = 0 \\ -1 - 1 = -2 \\ 1 - 3 = -2 \end{matrix}$$

$$R_3 \rightarrow R_3 - R_1$$

$$\begin{matrix} 1 - 1 = 0 \\ -2 - 1 = -3 \\ 0 - 3 = -3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -2 \\ 0 & -3 & -3 \end{bmatrix} \begin{matrix} R_3 \rightarrow (R_3 \times 2) - R_2 \times 3 \\ 0(2) - 3(-2) = 0 \\ -3(2) - 3(-2) = -6 + 6 = 0 \\ -3(2) - 3(-2) = -6 + 6 = 0 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{matrix} x + y + 3z = 0 \rightarrow 0 \\ -2y - 2z = 0 \rightarrow 0 \end{matrix}$$

$$\text{Rank}(A) = 2$$

$$\text{no. of variables} = 3$$

$\text{Rank}(A) < \text{no. of variables}$

$\therefore$ System of Equation is consistent
and have infinity solutions

$n - r$ arbitrary parameter in solution

$$3 - 2 = 1$$

arbitrary parameter in solution

consider z is arbitrary

let $z = k$, sub z in ②

$$-2y - 2(k) = 0$$

$$-2y = 2k$$

$$2y = -2k$$

$$y = -k$$

sub y, z in Equation ①

$$x + y + 3(z) = 0$$

$$x + (-k) + 3(k) = 0$$

$$x - k + 3k = 0$$

$$x + 2k = 0$$

$$x = -2k$$

$$\therefore x = -2k, y = -k, z = k$$

14Q)

$$\begin{aligned} 4x+2y+z+2w &= 0 \\ 6x+3y+4z+2w &= 0 \\ 2x+y+w &= 0 \end{aligned}$$

Sol:-

$$\begin{cases} 4x+2y+z+2w=0 \\ 6x+3y+4z+2w=0 \\ 2x+y+w=0 \end{cases} \text{ given}$$

Equation $\rightarrow$ matrix form $(Ax=0)$

$$A = \begin{bmatrix} 4 & 2 & 1 & 2 \\ 6 & 3 & 4 & 2 \\ 2 & 1 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 1 & 2 \\ 6 & 3 & 4 & 2 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

matrix form $\rightarrow$ row reduction form

$$\begin{bmatrix} 4 & 2 & 1 & 2 \\ 6 & 3 & 4 & 2 \\ 2 & 1 & 0 & 1 \end{bmatrix} \quad R_3 \leftrightarrow R_1$$

$$= \begin{bmatrix} 2 & 1 & 0 & 1 \\ 6 & 3 & 4 & 2 \\ 4 & 2 & 1 & 2 \end{bmatrix} \quad \begin{aligned} R_2 &\rightarrow R_2 - R_1 \times 3 \\ R_3 &\rightarrow R_3 - R_1 \times 2 \end{aligned}$$

$$\begin{aligned} 6-3(2) &= 6-6=0 \\ 3-3(1) &= 3-3=0 \\ 4-3(0) &= 4-0=4 \\ 2-3(1) &= 2-3=-1 \end{aligned}$$

$$R_3 \rightarrow R_3 - R_1 \times 2$$

$$4 - 2(2) = 4 - 4 = 0$$

$$2 - 2(1) = 2 - 2 = 0$$

$$1 - 2(0) = 1 - 0 = 1$$

$$3 - 2(1) = 3 - 2 = 1$$

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\text{Rank}(A) = 3$$

$$\text{no. of variables } (n) = 4$$

$$\therefore \text{Rank}(A) < \text{no. of variables } (n)$$

$\therefore$ The system of Equations is homogeneous and is consistent and infinitely many solutions

Solving

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$2x + y + w = 0 \rightarrow \textcircled{1}$$

$$4z + 4w = 0 \rightarrow \textcircled{2}$$

$$z + w = 0 \rightarrow \textcircled{3}$$

n -r arbitrary parameter in solution
 $4-3=1$, 1 arbitrary parameter in,
consider z is arbitrary

let $z=k$, sub in Equation (3)

$$2x+w=0$$

$$(k)+w=0$$

$$w=-k.$$

150) for what values of λ the given Equation will have a non solution

$$x+2y+3z=\lambda x$$

$$2x+3y+z=\lambda x$$

$$3x+y+2z=\lambda y.$$

sol:- The above set of Equation can be written

$$\text{as } \begin{bmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

i.e. $Ax=0$ where A is coefficient matrix
(1) will have a non-zero solution, if
the coefficient matrix A should have a
rank less than 3. If the rank of the
coefficient matrix is less than 3 then

we must have $|A| = 0$

$$\text{i.e., } \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_2 + R_3$

$$\begin{vmatrix} 6-\lambda & 6-\lambda & 6-\lambda \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

Taking common $(6-\lambda)$ from R_1

$$(6-\lambda) \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

Applying $C_2 \rightarrow C_2 - C_1$ $C_3 \rightarrow C_3 - C_1$

$$(6-\lambda) \begin{vmatrix} 1 & 0 & 0 \\ 3 & -\lambda & -1 \\ 2 & 1 & -\lambda-1 \end{vmatrix} = 0$$

Expanding along C_1 , we get

$$(6-\lambda)[(\lambda+2)(\lambda+1)+1] = 0 = \lambda(6-\lambda)(\lambda^2+3\lambda+3)$$

Either $6-\lambda=0$ or $\lambda^2+3\lambda+3=0$ $\lambda^2+3\lambda+3=0$

$$\lambda = \frac{-3 \pm \sqrt{9-12}}{2} \quad \lambda = \frac{-3 \pm \sqrt{-3}}{2}$$

which are imaginary

$\therefore \lambda = 6$ is the only real value of λ for which the system (1) has non-zero solution

160) find the value of 'a' so that the following system of homogeneous Equations have Exactly 2 linearly independent solutions

$$ax_1 - x_2 - x_3 = 0$$

$$-x_1 + ax_2 - x_3 = 0$$

$$-x_1 - x_2 + ax_3 = 0$$

Sol:-
$$\begin{bmatrix} a & -1 & -1 \\ -1 & a & -1 \\ -1 & -1 & a \end{bmatrix}$$

$$= a \begin{vmatrix} a & -1 \\ -1 & a \end{vmatrix} + 1 \begin{vmatrix} -1 & -1 \\ -1 & a \end{vmatrix} + 1 \begin{vmatrix} -1 & -1 \\ -1 & -1 \end{vmatrix}$$

$$a(a^2 - 1) + 1(-a - 1) + 1(1 - 1 - a)$$

$$a(a^2 - 1) + 1(-a - 1) + 1(1 - a)$$

$$= a^3 - a - a - 1 - 1 - a =$$

$$a^3 - 3a - 2$$

∴ roots are 2 and -1

2 linearly independent solutions are

$$a = 2, -1$$

170) Apply the test of rank to Examine if the following Equations are consistent

$$2x - y + 3z = 8$$

$$-x + 2y + z = 4$$

$$3x + y - 4z = 0$$

and if consistent, find the complete solution

Sol: $(A) = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$

$$(A/B) = \begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix} \quad R_2 \rightarrow 2R_2 + R_1$$

$$2(-1) + 2 = 0$$

$$2(1) + (-1) = 3$$

$$2(4) + (8) = 16$$

$$R_3 \rightarrow 2R_3 - 3R_1$$

$$2 \times (8) - 3 \times (2) = 0$$

$$2 \times (1) - 3 \times (-1) = 5$$

$$2 \times (-4) - 3 \times (3) = -17$$

$$2 \times (0) - 3 \times (8) = -24$$

$$[A/B] = \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 3 & 5 & 16 \\ 0 & 5 & -17 & 24 \end{bmatrix}$$

$$R_3 \rightarrow 3R_3 - 5R_2$$

$$3(16) - 5(0) = 0$$

$$3(5) - 5(3) = 0$$

$$3(-17) - 5(5) = -76$$

$$R(A) = 3 \quad R(A/B) = 3 \quad n = 3$$

$$\therefore R(A) = R(A/B) = n$$

It is consistent and unique solution

$$2x - 4y + 3z = 8$$

$$3y + 5z = 16$$

$$76z = 15$$

$$z = \frac{15}{76} \approx 2$$

$$\boxed{z = 2}$$

from Equation (2) $3y + 5(2) = 16$

$$3y + 10 = 16$$

$$3y = 16 - 10$$

$$3y = 6$$

$$y = \frac{6}{3} = 2 \quad \boxed{y = 2}$$

from Equation (1)

$$2x - (2) + 3(2) = 8$$

$$2x - 2 + 6 = 8$$

$$2x + 4 = 8$$

$$2x = 8 - 4 \Rightarrow \boxed{x = 2}$$

$$x = 2, y = 2, z = 2$$

(18) show that the Equations

$$-2x + y + z = a$$

$$x - 2y + z = b$$

$$x + y - 2z = c$$

have no solution unless $a + b + c = 0$,
in which case they have infinitely
many solutions find their solutions
 $a=1, b=1$ and $c=-2$.

Sol:- Given

$$-2x + y + z = a$$

$$x - 2y + z = b$$

$$x + y - 2z = c$$

$$(A) = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Where A is coefficient matrix

Let $C = (A/B)$ the augmented matrix

$$C = \begin{bmatrix} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & c \end{bmatrix}$$

applying $R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & 1 & -2 & c \\ 1 & -2 & 1 & b \\ -2 & 1 & 1 & a \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$(1) - (1) = 0$$

$$(-2) - (1) = -3$$

$$(1) - (-2) = 3$$

$$(b) - (c) = b - c$$

$$R_3 \rightarrow R_3 + 2R_1$$

$$-2 + 2(1) = 0$$

$$1 + 2(1) = 3$$

$$1 + 2(-2) = -3$$

$$a + 2(c) = a + 2c$$

$$\sim \begin{bmatrix} 1 & 1 & -2 & c \\ 0 & -3 & 3 & b - c \\ 0 & 3 & -3 & a + 2c \end{bmatrix}$$

$$R_3 \rightarrow R_3 + R_2$$

$$0 + 0 = 0$$

$$3 + (-3) = 0$$

$$(-3) + (3) = 0$$

$$(a + 2c) + (b - c) = a + b + c$$

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$a + b + c \neq 0$ have

$$P(A) = 2$$

$$P(C) = 3$$

$$P(A) < P(C)$$

Hence the Equations are inconsistent if $a+b+c \rightarrow$ we have

$$p(A) = p(C) \rightarrow 2 < 3 \text{ (no. of unknowns)}$$

Hence the Equations are consistent and have infinitely many solutions obtained by giving arbitrary value Eq (1) reduces to the form.

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} c \\ a-c \\ 0 \end{bmatrix}$$

$$x + y - 2z = c$$

$$-3y + 3z = b - c$$

$$\text{given } b=1 \quad c=-2$$

substituting in (2) & (3) we have

$$x + y - 2z = -2$$

$$-3y + 3z = -2$$

Taking $z=k$ we have $y=k-1$

$$x = -2 - y + 2z$$

$$= -2 - k + 1 + 2k$$

$$\boxed{x = k-1}$$

$$-3y + 3k = 3$$

$$-3y = 3 - 3k$$

$$-3/3 y = 3(k-1)/3$$

$$\boxed{y = k-1}$$

21Q) Solve the following system of Equations by Gaussian Elimination method

$$x_1 + 2x_2 - 3x_3 = 3$$

$$2x_1 - 2x_2 + 3x_3 = 2$$

$$3x_2 - x_2 + 2x_3 = 1$$

$$x_1 - x_2 + x_3 = -1$$

$$(A) = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 2 \\ 3 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad |B| = \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \end{bmatrix}$$

$$AX = B$$

Let $C = (A, B)$ the augmented matrix

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 3R_1$$

$$(3) - (1) \times 3 = 0$$

$$(-1) - (2) \times 3 = -7$$

$$(2) - (-1) \times 3 = 5$$

$$(1) - (3) \times 3 = -8$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$2 - 2(1) = 0$$

$$-2 - 2(2) = -6$$

$$3 - 2(-1) = 5$$

$$2 - 2(3) = -4$$

$$R_4 \rightarrow R_4 - R_1$$

$$1 - 1 = 0$$

$$-1 - 2 = -3$$

$$1 - (-1) = 2$$

$$-1 - 3 = -4$$

$$= \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -9 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_3$$

$$0 - 0 = 0$$

$$-7 - (-6) = -1$$

$$5 - (5) = 0$$

$$-8 - (-4) = -4$$

$$R_3 \rightarrow R_3 - 6R_2$$

$$0 - (0)6 = 0$$

$$-6 - (-7)6 = 0$$

$$5 - (-5)6 = 5$$

$$-4 - (-8)6 = 20$$

$$R_4 \rightarrow R_4 - 3R_2$$

$$0 - (0) \times 3 = 0$$

$$-3 - (-7) \times 3 = 0$$

$$2 - (5) \times 3 = -2$$

$$-4 - (-8) \times 3 = 8$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -1 & 0 & -4 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 2 & 8 \end{bmatrix}$$

$$R_2 \rightarrow R_2 \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 2 & 8 \end{bmatrix}$$

$$R_3 \rightarrow (1/5)R_3$$

$$\frac{1}{5} 0 = 0$$

$$\frac{1}{5} 0 = 0$$

$$\frac{1}{5} 20 = 4$$

$$\frac{1}{5} 20 = 4$$

$$R_4 \rightarrow R_4 (1/2)$$

$$0 \cdot 1/2 = 0$$

$$0 \cdot 1/2 = 0$$

$$2 \cdot 1/2 = 1$$

$$8 \cdot 1/2 = 4$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_3$$

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$1 - 1 = 0$$

$$4 - 4 = 0$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The rank of coefficient matrix A is 3
and the rank of augmented matrix is also 3

$$\rho(A) = \rho(C) = 3$$

= no. of unknowns.

Therefore Equations are consistent and
they have a unique solution Equation

(1) reduces to the form.

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 3 \\ 4 \\ 4 \\ 0 \end{bmatrix}$$

$$x + 2y - z = 3 \quad \text{--- (2)}$$

$$\boxed{y = 4} \quad \text{--- (3)}$$

$$\boxed{z = 4} \quad \text{--- (4)}$$

Equation (1)

$$x + 2(4) - 4 = 3$$

$$x + 8 - 4 = 3$$

$$x + 4 = 3 \Rightarrow x = 3 - 4$$

$$\boxed{x = -1, y = 4, z = 4} \quad \boxed{x = -1}$$